B.TECH
(SEM - I) THEORY EXAMINATION 2018-19
ENGINEERING MATHEMATICS - I

Time: 3 Hours   *Total Marks: 100*

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. **Attempt all questions in brief.**   **2 x10 = 20**

    a. If $u(x,y) = (\sqrt{x} + \sqrt{y})^5$, find value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.

    b. Find all symmetry of the curve $y^2(2a - x) = x^3$.

    c. If $x = r\cos\theta, y = r\sin\theta$ find value of $\dfrac{\partial(x,y)}{\partial(r,\theta)}$.

    d. Find the percentage error in the area of an ellipse when an error of +1% is made in measuring the major and minor axes.

    e. Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.

    f. Show that the matrix $\begin{bmatrix} 2 & 3-4i \\ 3+4i & 5 \end{bmatrix}$ is Hermitian.

    g. Change the order of integration in $\displaystyle\int_0^\infty \int_x^\infty f(x,y)\,dy\,dx$.

    h. Use triple integral to find volume of a sphere of unit radius.

    i. Find $grad\ \varphi$ when φ is given by $\varphi = 3x^2y - y^3z^2$ at the point $(1,-2,-1)$.

    j. If $\vec{r} = x\,\hat{i} + xy\,\hat{j} + x\,\hat{k}$ find $div\vec{r}$ and $curl\vec{r}$.

SECTION B

2. **Attempt any *three* of the following:**   **10 x 3 = 30**

    a. If $y = e^{a\sin^{-1}x}$, find $(y_n)_0$.

    b. If $u^3 + v^3 + w^3 = x + y + z, u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$, then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.

    c. Diagonalise the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$.

    d. Evaluate $\displaystyle\iint_R (x^2 + y^2)\,dx\,dy$ where R is the region in the first quadrant bounded by $x^2 - y^2 = a, x^2 - y^2 = b, 2xy = c, 2xy = d, 0 < a < b, 0 < c < d$.

e. Verify Green's theorem for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by the lines $x = 0, y = 0, x + y = 1$.

SECTION C

3. **Attempt any *one* part of the following:** 10 x 1 = 10

(a) Expand $\tan^{-1} \dfrac{y}{x}$ in the neighbourhood of $(1,1)$ upto and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$ approximately.

(b) Trace the curve $r = a(1 + \cos \theta)$.

4. **Attempt any *one* part of the following:** 10 x 1 = 10

(a) In estimating the number of bricks in a pile which is measured to be $(5m \cdot 10m \cdot 5m)$, the count of bricks is taken as 100 bricks per m^3. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of the bricks is Rs. 2000 per thousand bricks.

(b) Find the volume of largest rectangular parallelopiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

5. **Attempt any *one* part of the following:** 10 x 1 = 10

(a) Determine the values of λ and $\propto$ such that the system $2x - 5y + 2z = 8$, $2x + 4y + 6z = 5, , x + 2y + \lambda z = \propto$

(b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, Hence compute A^{-1}.

6. **Attempt any *one* part of the following:** 10 x 1 = 10

(a) Determine the area of region bounded by the curves $xy = 2, 4y = x^2, y = 4$.

(b) Find the volume of the solid surmounted by the surface $\left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3} + \left(\dfrac{z}{c}\right)^{2/3} = 1$.

7. **Attempt any *one* part of the following:** 10 x 1 = 10

(a) Show that the vector field $\vec{F} = \dfrac{\vec{r}}{r^3}$ is irrotational as well as solenoidal.

(b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.